Tutorial Quiz 2018

MATH1013 - Mathematics and Applications 1

Tutorial Quiz 8
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 15 minutes

Student Name: _____
University ID: _____

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
3	2	15

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 7 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Linear Algebra

Question 1

Suppose $S : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear operator such that

$$S\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}3\\5\end{bmatrix} \quad \text{and} \quad S\left(\begin{bmatrix}0\\4\end{bmatrix}\right) = \begin{bmatrix}1\\4\end{bmatrix}.$$

Evaluate $S\left(\begin{bmatrix}1\\-2\end{bmatrix}\right)$.

[3 marks].

Question 2

Determine the value(s) of $h \in \mathbb{R}$ such that

$$\text{span}\left\{\begin{bmatrix}1\\3\\4\\5h+\sqrt{3}\end{bmatrix}, \begin{bmatrix}2+\sqrt{5}\\4-\frac{\pi}{1+\sqrt{3}}\\1-\sqrt{e}\\3h+1\end{bmatrix}\right\} = \mathbb{R}^4.$$

[1 marks].

Question 3

Determine whether the statement: *The zero vector* $\mathbf{0}$ *is contained in every vector space* is a true statement. Justify your answer. [2 marks].

Calculus

Question 1

Evaluate the following derivatives.

(a)

$$\frac{d}{dx}\left(x^{\cos x}\right).$$

[3 marks].

(b)

$$\frac{d}{dx}\left(\int_{-x}^{\frac{1}{3x+1}-4\sin(x)} \sec^3\left(\sqrt{s-\log_e(s)+10\pi s}\right) ds\right).$$

[3 marks].

(c)

$$\frac{d}{dx} x^{\tan(x)}.$$

[3 marks].

Bonus Question

Differentiate the function

$$h(x) = \sin^{-1}\left(\frac{f(x)}{\sqrt{g(x)}}\right),$$

where f and g are differentiable everywhere and $g(x) > 0$ for all $x \in \mathbb{R}$. [3 marks].