

Tutorial Quiz 2018

# MATH1014 - Mathematics and Applications 2

## Tutorial Quiz 10 Calculus and Linear Algebra

Reading time: 1 minute  
Writing time: 9 minutes

Student Name: \_\_\_\_\_  
University ID: \_\_\_\_\_

### Question and Answer Book

#### Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
2	2	15

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to collaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

#### Materials supplied

- Question and answer booklet of 5 pages.
- Working space is provided throughout the booklet.

#### Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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### Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### Question 1

(5 marks). For each statement, decide whether it is always true (**T**) or sometimes false (**F**) and write your answer clearly next to the letter before the statement.

- (a) An invertible matrix is diagonalisable.
- (b) A diagonalisable matrix is invertible.
- (c) If  $A$  is similar to a diagonalisable matrix, then  $A$  is diagonalisable.
- (d) The rank of a matrix is equal to the number of nonzero eigenvalues.
- (e) If  $A$  and  $B$  are similar matrices, then  $\det(A) = \det(B)$ .

## Question 2

(4+3+3 = 10 marks).

(a) The matrix  $A$  below has been factored in the form  $A = PDP^{-1}$ , with  $D$  diagonal.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

Find the eigenvalues of  $A$  and a basis for each eigenspace.

(b) Explain why the matrix

$$A = \begin{bmatrix} 5 & 1 & 2 \\ 0 & -1 & 17 \\ 0 & 0 & 6 \end{bmatrix}$$

is diagonalisable.

(c) Show that the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

is not diagonalisable.

**END OF TUTORIAL QUIZ**