

Tutorial Quiz 2018

MATH1013 - Mathematics and Applications 1

Tutorial Quiz 5 Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 10 minutes

Student Name: _____
University ID: _____

Question and Answer Book

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
4	4	13

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to collaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied

- Question and answer booklet of 4 pages.
- Working space is provided throughout the booklet.

Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Linear Algebra

Question 1

Let $v_1, v_2, v_3 \in \mathbb{R}^3$ be the vectors given by

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}, \quad v_3 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}.$$

Determine the value(s) of $h \in \mathbb{R}$ such that v_2 lies in the span of v_1 and v_3 . [3 marks].

Calculus

Question 1

Show that for all $0 < x < \frac{\pi}{2}$,

$$\tan x > x.$$

[Hint: Consider $f(x) = \tan x - x$.]

[3 marks].

Question 2

Describe the graph of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ which satisfies:

- (i) $f \in \mathcal{C}^2(\mathbb{R})$ (f is twice-differentiable for all $x \in \mathbb{R}$).
- (ii) The derivative of f crosses the x -axis at $x = 3$ and $x = -2$.
- (iii) $f''(3) = 4$ and $f''(-2) = -5$.

[3 marks].

Turn Over.

Question 3

Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = x|x|,$$

is differentiable at $x = 0$ and evaluate $f'(0)$, where f' denotes the derivative of f . [4 marks].

END OF TUTORIAL QUIZ

Questions sourced from:

- † Broder, K. *An Invitation to Analysis - A First Course in Mathematics*, Akadem, (2018).
- † Broder, K. *An Introduction to Analysis - A First Course in Analysis*, Akadem, (2016).