Tutorial Quiz 2018

MATH1014 - Mathematics and Applications 2

Tutorial Quiz 3
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 12 minutes

Student Name: ________________
University ID: ________________

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	10

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 5 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let V be an $\mathbb{R}$-vector space. That is, V is a set such that

(i) (Vector addition). $v_1 + v_2 \in V$ for all $v_1, v_2 \in V$.

(ii) (Scalar multiplication). $\lambda v \in V$ for all $v \in V$, $\lambda \in \mathbb{R}$.

(a) Determine which of the following sets are vector spaces. No justification is required if they are. If they are not, justify exactly which condition fails.

(i) $\mathbb{R}^n$. (Euclidean n-space). [1 mark].

(ii) $\mathbb{C}$. (The complex plane). [1 mark].

(iii) $\mathbb{N}$. (The natural numbers). [1 mark].

(iv) $\mathbb{Z}$. (The integers). [1 mark].

(b) Show that every vector space V contains the zero vector.

[Hint: Use the definition given at the start of the Question]. [2 marks].

(c) Suppose that V_1 is a subset of V_2, where V_2 is an $\mathbb{R}$-vector space. What does it mean for V_1 to be a *subspace* of V_2? [1 mark].

(d) Let $\mathscr{C}(\mathbb{R})$ denote the space of continuous functions $f : \mathbb{R} \longrightarrow \mathbb{R}$. It is clear that $\mathscr{C}(\mathbb{R})$ forms an $\mathbb{R}$-vector space. Let $\mathscr{C}^1(\mathbb{R})$ denote the space of continuously differentiable functions $f : \mathbb{R} \longrightarrow \mathbb{R}$. In other words, elements of $\mathscr{C}^1(\mathbb{R})$ are functions which are differentiable, and whose derivative is continuous. Show that $\mathscr{C}^1(\mathbb{R})$ is a subspace of $\mathscr{C}(\mathbb{R})$. [2 marks].

(e) Provide an example of a function which lies in $\mathscr{C}(\mathbb{R})$, but not in $\mathscr{C}^1(\mathbb{R})$. [1 mark].