Tutorial Quiz 2018

MATH1013 - Mathematics and Applications 1

Tutorial Quiz 6
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 10 minutes

Student Name: _____
University ID: _____

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	11

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 4 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Intructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

The terms *linear transformation* and *linear operator* are equivalent.

Question 1

Let $\mathscr{C}([0,1])$ denote the space of continuous functions on $[0,1]$. The operator $T : \mathscr{C}([0,1]) \longrightarrow \mathbb{R}$ defined by

$$f \longmapsto \int_0^1 f(x)dx$$

provides an example of a linear operator.

a. Define *linear transformation.* [2 marks].

b. Consider the following computation.

$$\begin{aligned}\int_0^1 x + 4x^2 dx &= \int_0^1 x dx + \int_0^1 4x^2 dx \\ &= \int_0^1 x dx + 4\int_0^1 x^2 dx = \frac{11}{6}.\end{aligned}$$

Detail exactly where the linearity of T is used in the above computation. [2 marks].

Question 2

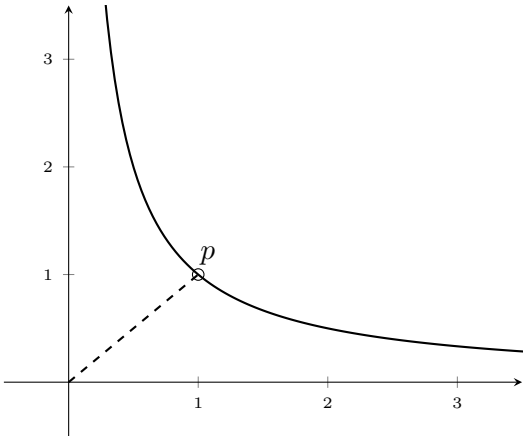
Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear operator defined by

$$T(x_1, x_2, x_3) = (x_1 + 4x_2 + 14x_3, 2x_1 + 9x_2 + 31x_3, x_1 + 5x_2 + 17x_3).$$

Write down the standard matrix for this transformation. [3 marks].

Question 3

Let X be the curve $xy = 1$ which is sketched below.



Determine the point $p \in X$ which is closest to the origin. [4 marks].