LESSONS FROM TEACHING MATH2305

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1. Overview of the course

In Semester 1 of 2022, I lectured MATH2305/MATH6405 – Mathematics and Applications 1, at the Australian National University. As detailed on the course webpage, the topics to be covered in the course included: Curves and surfaces in three dimensions; parametric representations; curvilinear coordinate systems; Surface and volume integrals; use of Jacobians; gradient, divergence and curl; identities involving vector differential operators; the Laplacian; Green's and Stokes' theorems.

The course consisted of 205 students, with the student demographic segregating into $\sim 65\%$ engineering, $\sim 20\%$ finance, $\sim 5\%$ mathematics, $\sim 10\%$ other. The mode of delivery consisted of four lectures ($4 \times 50 = 200$ minutes) per week, given in person, Wednesday 9:05am–9:55am and 10:05am–10:55am; and Thursday 9:05am–9:55am and 10:05am–10:55am.

Because of the hybrid nature of the pandemic at the time, online recordings were made available to the students that were not in Australia during the semester, or did not attend the lectures for safety or quarantine related reasons. To meet the hybrid learning requirement I wrote a very thorough textbook reference that is available here: Vector Calculus Book, and recorded the board lectures I gave, posting them here: Lecture recordings.

Summary of the Predominant Philosophies on Teaching this course. There is the computation-focused treatment, primarily catering to engineering students, which avoids any systematic development of the theory and presents the results in a very physical manner. This is the direction taken, for instance, in J. Stewarts' *Calculus* [4], and does a very good job at treating the subject from this lens. At this point, the reader is likely to interject with, *well, engineering students are the primary audience.* There is a pedagogical drawback to this approach, however, that not only afflicts the pure mathematics students, but also impacts the target audience; namely, the engineering students. The approach taken in Stewarts' book side-steps the systematic theory of differential forms that has been developed over the last century, in favor of the less intimidating ad-hoc approach involving the grad vector, the cross product, and so on. The price one pays for this is that, despite each individual lecture being rather straightforward and elementary, by the end of the teaching semester, students are left with a vast number of disparate and unrelated collection of facts.

On the other side, the high-brow approach of doing everything via forms, the enlightened approach taken, for instance, most notably in H. M. Schey's *Div, Grad, Curl, and all that*, the audience is evidently those who have seen the computation-focused approach and wish

to achieve enlightenment through the theory of differential forms. This is not a criticism of Schey's book – the book has the sub-heading: *an informal text on vector calculus*. Harold M. Edwards – *Advanced Calculus – A Differential Forms Approach* [2] is a more appropriate illustration of this second class. Here, the theory is put first, but the intended audience is strange, at best. The amount of mathematical maturity required to digest [2] is likely only held by those students who have taken much more advanced courses than vector calculus.

Aim of the Course. The aim of the course of lectures I gave at ANU in Semester 1 of 2022 was to provide an appropriate middle ground. The intention was to develop the theory properly, such that students walk away from the course with a coherent picture of the subject. Further, the material was presented in a digestable manner. The first chapter may be labeled naive, but there is never a moment in the text where a false statement is given. Further, students who wish for a more detailed account of the developments are encouraged to see the appendix.

2. Some Lessons

Effective Teaching Practices. During the delivery of the lectures, a number of approaches to the delivery seemed to work effectively:

(1) Any theoretical explanation, new concept, or definition, was followed immediately by a series of explicit and non-trivial examples. This allowed students to build intuition through computation.
(2) Repetition. I repeated the central message of the course, i.e., the different guises of Stokes' theorem, many times. I did not wait until the end of the course to do a review of what we had learned so far in the course, I did this as often as I could.
(3) I followed the book very thoroughly. Since I had access to both a big projector screen that could project the view of my laptop, and a whiteboard under this screen, I used both. I projected the relevant component of the lecture notes so that students that wanted to copy down definitions in the lecture, had precise statements to copy down. But for the delivery of the lecture, I wrote on the board as if there was no projector above me.
(4) I maintained a guiding narrative. The students knew that we were developing a theory of vector calculus. They were told from the very beginning what this would entail, i.e., a differentiation theory, an integration theory, etc. Hence, if the students never did another mathematics course again, they can still achieve a global, and meaningful, understanding of what vector calculus is.
(5) I spoke to the students directly in the lecture. I lectured the class of 100 or so students that attended the lecture in the same manner that I would teach to a class of 20 students in a tutorial.

Non-Effective Teaching and Assessment Practices. The following were problems of the course that in future courses, I would correct:

(1) Recording quality of the lectures. I would recommend buying a reasonably good camera to record better quality videos of the white board. This is a much better experience for both the students and the lecturer than simply delivering from slides.
(2) Handwritten exams. Due to the hybrid nature of delivery, the exam had to be online. Most of the negative feedback from the students concerning the course was focused on this aspect. This was, unfortunately, out of my control.
(3) Given that the lecture notes were being written and extended throughout the semester, I had little time to include answers to various exercises. This would have been helpful to the students, and in the absence of these in my own notes, I would have provided a supplementary reference or references that had exercises with answers.

3. Feedback from the students (ANU SELT Reviews)

At the end of the semester, the students are encouraged to submit

What were the strengths of the teacher's approach?

- Subject interest, enthusiasm, organisation of information and supporting material.
- Organisation, taking student input and staying behind after lectures to answer questions.
- A very engaging, if eccentric, lecturer. Clearly was passionate about the subject matter and communicated key concepts well.
- Easy to follow explanations of topics backed up with examples and notes helped my learning extremely well, felt like Kyle wanted to teach the maths instead of just throwing the content at you.
- Kyle was very enthusiastic and was able to engage with students throughout the course.
- Kyle was a very engaging lecturer who taught his content very effectively.
- Kyle was an extremely engaging, knowledgeable, and humorous lecturer who was always approachable for a variety of questions and problems. His lectures were delivered very well and followed the ODE notes very closely, which gave his portion a well-defined and easy-to-follow structure. His inclusion of extended topics in the VC notes was a welcomed addition and was a very helpful read for students considering higher-level study in this field.
- Clear and concise teaching. Happy to talk to students after the lectures about the course content and further content and general interest in mathematics. All course content was covered in lectures to a good degree.
- Easy to understand, organised and made the course enjoyable.

- Good lecturer.
- Kyle had a unique approach to the subject of vector calculus that made the content really easy to understand and follow from week to week. He was also very clearly enthusiastic about the topic that he was teaching.
- Creating the building blocks for and hinting at more advanced mathematics (differential geometry), vector calculus textbook/notes were extremely useful.
- Kyle's live lectures were very engaging and helpful. He used lots of examples and taught with a good pace, and allowed us to ask questions. He sometimes made mistakes or communicated things unclearly but he always fixed these errors afterwards. This first part of the course was excellent, there was a clear relationship between the lectures, weekly quizzes and workshops.
- Great lectures and is a great teacher explains things very clearly and succinctly. The in person lectures were awesome.
- Lectures were good and textbook written to follow content was great especially if you missed anything in the lectures
- Taught really difficult concepts such as hodge star operator and triple integrals with ease and made it really easy to understand. Charismatic and very personable.
- The VC textbook is fantastic, the diagrams and problems are great and I found it was an amazing resource. I thought the content being taught was really good, and Kyle was a fairly engaging lecturer.
- I loved Kyle's lectures, they were super engaging and useful. However, there was little feedback or help provided outside of these, including only a limited number of answers for textbook and revision questions which made it difficult to test your understanding of difficult topics and almost impossible to learn outside of the lecture material. Because the approach of his textbook was very different to internet and previous course materials, there was little opportunity to find help elsewhere including from the drop-in centre.
- Humorous.
- Really clear and engaging lectures.
- Kind.
- One of the best lecturers I have had. They are very engaging and made the content enjoyable.
- Going through examples in class and taking a slower paced approach to a difficult subject allowed me to do well.
- Genius.
- He was a very engaging lecturer.

- The course notes were extremely useful, if a little dense sometimes. I found they were very helpful in my understanding of the concepts.
- Kyle was energetic, personable, communicated very clearly about what the key concepts we needed to learn were (e.g. with a summary at the end of each lecture), and spent time after lectures discussing extensions of the content. During lectures he pointed out further topics and relations of the content to other areas of mathematics, which was interesting. He did a decent job simplifying abstract ideas to make them more approachable (e.g. differential forms, Hodge-star, the meaning of 'simply connected', etc.).
- His humour made the lectures very engaging and enjoyable He was quite easily one of the best lecturers I've been taught by.
- Great learning resources that paralleled the course material
- Really enjoyed Kyles approach to VC, it was the most intuitive that I have come across, and made the subject very easy to understand.
- Everything (especially telling jokes in classes).
- Very good presentation style, was open to student feedback.
- Kyle was a good lecturer, his content was easy to follow, and his textbook was well produced.
- Very understanding of illness. Generally engaging in lectures.
- He showed up on time.
- Really good teacher, seemed interested in content.
- Good teaching and warm-hearted.
- In the few lectures I watched, he seemed like quite an engaging lecturer. In addition, the short YouTube videos were great.
- He made difficult concepts easy to understand and he was well knowledgeable in his field.
- Kyle was a great teacher, and his textbook was great as well. He made the subject fun and engaging.
- Explained concepts well and tied them together nicely. Lecture notes were great too.
- Kyle developed an excellent textbook which explained things quite well.

Do you have any suggestions about how the teacher's approach could be improved?

- None.
- The assessment on Kyle's section of the course was poor - it is not appropriate to assess maths without allowing students to show marking.
- I think that some greater structure in the lectures would be valuable: a clear focus on outlining the theoretical foundations of the week's topic and then diving into some

examples. Also, I felt as though a lot of the examples we did were too easy, we rarely went through exam level questions in class.

- Some better organisation with the lectures (especially the notes) might be useful.
- Kyle was not very responsive to emails. His recordings of the lectures were of fairly low quality since they were recorded on a phone and it was often difficult to see the board or hear him speak, which may have been unfair to students who were not physically able to attend the lectures.
- Could be bit more organised.
- The exam questions didn't cover what he said it would cover and was not in line with the content of the course. Also was very annoying that the mid-semester practice exam was not given solutions.
- Maybe some more applications.
- N/A.
- I found the lecture pacing a bit odd (sometimes it felt like Kyle was just trying to kill time in lectures), but I also understand this is his first time lecturing the course. Perhaps another method of filming lectures could be considered (to show the lecture slides/text book as well as what is being written).
- Making answers to practise and textbook questions would be great, and replying to emails where possible.
- No issues, keep doing what you were doing.
- He could have a better approach to making videos of the lectures as the recordings were unreliable.
- Kyle was sometimes disorganised (e.g. with recording lectures), which could be better.
- Maybe making sure the solutions in the textbook are correct.
- Communication paths between the teacher and students need to be strengthened.
- I wish that there was more worked examples offered in the notes/textbook, particularly for surface integrals and stokes theorem. I would have liked more time spent going over them, as that was something I struggled with. I also felt that there was some aspects in the exam that was not covered in the lectures, e.g the curve that maximises the line integral.
- No.
- Give examples after introducing a concept instead of before, makes revising easier if theorems aren't given between examples.
- Lecture recordings weren't good.
- I think uploading the videos on time and in a watchable state would be helpful in the future, I did not like watching them so I just read the notes/textbook.

- The lecture recordings were a serious issue. Due to unavoidable class clashes, I could not make the lecture times. In the Vector Calc half of the course, the lecture recordings made it extremely difficult to see any of the writing on the board and the audio was terrible. The mid-semester exam format was awful. I would be getting 0/4 for questions where I had pages of working out, since it was a wattle quiz that provided no opportunity to demonstrate understanding of the question. Every other maths course I have done has had no problem sending out regular papers and getting people to scan and upload their solutions. The use of a wattle quiz honestly just felt like laziness on the lecturers' part which massively impacted the student's ability to perform well in the assessments. The class reps provided feedback that could have massively improved the course, yet they were repeatedly ignored and overuled by the lecturers. This makes it even more sad that this disastrous course organisation could have been avoided or fixed.
- Needs to be more communication with who is writing the exams. The structure and content gaps were large (e.g. the projection map question in the midsem was never covered).
- I think that a flipped class room with more YouTube like videos would be awesome.
- Should proofread the textbook, reply emails more frequently.
- Lecture recordings were terrible. Kyle said they were of an acceptable quality, but realistcally, you could barely see the whiteboard at times and the audio wasn't much better. Remote students were definitely at a disadvantage because of this.
- He should've proofread the textbook at the start of the course to fix the errors in there. There was a general lack of communication between the lecturer and the students. This led to a very fractured walkthrough of concepts taught in the lectures versus the textbook. Furthermore, the mid-semester exam really highlights the lack of communication as no support or explanation was provided to the later questions, which were not relevant to the course material.

References

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