Tutorial Quiz 2018

MATH1013 - Mathematics and Applications 1

Tutorial Quiz 9
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 15 minutes

Student Name: _____
University ID: _____

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 7 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Linear Algebra

Question 1

Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the operator given by $\mathbf{v} \longmapsto A\mathbf{v}$, where

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 8 \end{bmatrix}.$$

(a) Define the *Null space* of a linear operator. [1 mark].

(b) Compute the null space of T. [2 marks].

(c) Determine, with justification, whether T is one-to-one. [1 mark].

(d) For an arbitrary vector $\mathbf{b} \in \mathbb{R}^3$, determine whether the system

$$A\mathbf{x} = \mathbf{b}$$

always admits a solution. [2 marks].

(e) Hence, or otherwise, determine whether T is onto. [1 mark].

(f) Hence, or otherwise, determine the rank of T. [1 mark].

Calculus

Question 1

Let f be the function defined by

$$f(x) = \tan^{-1}(\sqrt{x+4}) + \sec(x).$$

Evaluate $f'(x)$. [3 marks].

Question 2

Evaluate

$$\frac{d}{dx}\left(\int_0^1 \exp(-\tan^{-1}(\xi^2))d\xi\right).$$

[2 marks].

Question 3

Evaluate the integral

$$\int \frac{\log_e(x)}{x} dx.$$

[3 marks].

End of Tutorial Quiz