Tutorial Quiz 2018

MATH1013 - Mathematics and Applications 1

Tutorial Quiz 7
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 10 minutes

Student Name: _____
University ID: _____

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
3	2	15

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 7 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Linear Algebra

Question 1

Recall that an operator $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is *linear* if

(i) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$,

(ii) $T(c\mathbf{u}) = cT(\mathbf{u})$ for all $\mathbf{u} \in \mathbb{R}^n$ and $c \in \mathbb{R}$.

(a) Use the definition to determine whether $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is linear, where

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 4 \\ x + 6 - 2y \end{bmatrix}.$$

[3 marks].

(b) Suppose that $S : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear transformation, such that

$$S\left(\begin{bmatrix}0\\3\end{bmatrix}\right) = \begin{bmatrix}1\\4\end{bmatrix} \quad \text{and} \quad S\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}3\\5\end{bmatrix}.$$

(i) Find $S\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$. (Show working). [2 marks].

(ii) Find $\mathbf{A}$, the standard matrix of S. [1 mark].

Calculus

Question 1

Evaluate the following derivatives.

(a)

$$\frac{d}{dx}\left(\int_0^4 \frac{1}{\sqrt{\log_e(|x|+\sec^2(x)+10)+\pi}} e^{-\sin(x)\tan^{-1}(x^3+1)}dx\right).$$

[Hint: *You have to think*[1].] [3 marks].

(b)

$$\frac{d}{dx}\left(e^{\int_1^x s ds}\right).$$

[3 marks].

[1] Sorry about that...

(c)

$$\frac{d}{dx}\left(x^{\cos(x)}\right).$$

[3 marks].

Bonus Question

Show that

$$\log_a(N)\log_b(N) + \log_b(N)\log_e(N) + \log_c(N)\log_a(N) = \frac{\log_a(N)\log_b(N)\log_c(N)}{\log_{abc}(N)}.$$

[4 marks].