Tutorial Quiz 2018

MATH1014 - Mathematics and Applications 2

Tutorial Quiz 6
Calculus and Linear Algebra

Reading time: 1 minute
Writing time: 12 minutes

Student Name: ________________
University ID: ________________

Question and Answer Book

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

Materials supplied
- Question and answer booklet of 5 pages.
- Working space is provided throughout the booklet.

Instructions
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions
Answer **all** questions in the space provided. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Denote by $\mathrm{M}_{n\times n}(\mathbb{R})$ the set of all $n \times n$ matrices with real entries. For a matrix A, we denote by $\sigma(A)$ the set of all eigenvalues of A. This is commonly referred to as the *spectrum of* A.

Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \in \mathrm{M}_{2\times 2}(\mathbb{R})$.

(a) Determine $\sigma(A)$. [1 mark].

(b) Determine $\sigma(B)$. [1 mark].

(c) Suppose that $0 \in \sigma(M)$ for some $M \in \mathrm{M}_{n\times n}(\mathbb{R})$. Determine whether the equation

$$M\mathbf{x} = 0$$

will have only the trivial solution $x = (0, ..., 0) \in \mathbb{R}^n$. [2 marks].

(d) Determine, with justification, whether a matrix $M \in \mathrm{M}_{n\times n}(\mathbb{R})$

(i) always has at least 1 eigenvalue. [1 mark]

(ii) always has at least n eigenvalues. [1 mark]

(iii) always has a real eigenvalue. [1 mark]

(iv) can have an infinite number of eigenvalues. [1 mark]

(e) Provide an example of a matrix $J \in \mathrm{M}_{2\times 2}(\mathbb{R})$ with no eigenvectors. [2 marks].

Question 2

A function $f : \mathbb{R} \to \mathbb{R}$ is said to be *(real) analytic at* $p \in \mathbb{R}$ if there exists an interval $U_\varepsilon = (p-\varepsilon, p+\varepsilon) \subseteq \mathbb{R}$ and a power series

$$\sum_{k=0}^{\infty} c_k (x-p)^k, \qquad c_k = \frac{1}{k!}\frac{d^k f}{\partial x^k}$$

which converges on U_ε and is equal to f on U_ε. If this holds for all $p \in G$, where $G \subseteq \mathbb{R}$, we say that f is analytic on G. Determine whether the following functions are analytic and if they are, state the domain on which they are analytic.

`In other words, a (real) analytic function is a function which admits a Taylor expansion with remainder being identically zero.`

(i) e^x. [1 mark]

(ii) $\cos(x)$. [1 mark]

(iii) A polynomial of degree 5. [1 mark]

(iv) The determinant function $\det : \mathrm{M}_{n\times n}(\mathbb{R}) \longrightarrow \mathbb{R}$. [1 mark]

(v) $\dfrac{1}{(x-3)^2}$. [1 mark]

END OF TUTORIAL QUIZ