**Tutorial Quiz 2018**

# MATH1014 - Mathematics and Applications 2

## Tutorial Quiz 5
## Calculus and Linear Algebra

**Reading time: 1 minute**
**Writing time: 12 minutes**

**Student Name:** ________________
**University ID:** ________________

## Question and Answer Book

**Structure of Book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 2 | 2 | 11 |

- Students are NOT permitted any calculators or notes during the quiz.
- Students are NOT permitted to colaborate in any form during the quiz. Any signs of collaboration or cheating will result in a nullified score and the course convenor will be informed of any academic misconduct.

**Materials supplied**
- Question and answer booklet of 5 pages.
- Working space is provided throughout the booklet.

**Instructions**
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the space provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

## Question 1

For each statement, decide whether it is always true (**T**) or sometimes false (**F**) and write your answer clearly next to the letter before the statement.

(a) The set of all $2 \times 2$ matrices with nonzero determinant with real entries is an $\mathbb{R}$-vector space.

[1 mark].

(b) If $V_1$ and $V_2$ are two $\mathbb{R}$-vector spaces, then $V_1 \cup V_2$ is a vector space. [1 mark].

(c) The intersection of three equations in $\mathbb{R}^3$ describes a point in $\mathbb{R}^3$. [1 mark].

(d) The set

$$\mathcal{H} = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : ab < 0 \right\}$$

is a subspace of $\mathbb{R}^2$. [1 mark].

(e) The linear map $T : \mathscr{C}^1(0,1) \to \mathscr{C}[0,1]$ defined by

$$T(f) = \frac{df}{dx}$$

is onto[1]. [1 mark].

(f) Every function $f : \mathbb{R} \to \mathbb{R}$ has a power series representation. [1 mark].

(h) Every polynomial in $\mathbb{P}_4$ may be identified with a vector in $\mathbb{R}^5$. [1 mark].

(i) The integral

$$\int_0^1 \frac{\cos(x) + 1}{x} dx$$

converges. [1 mark].

(j) Let $f : (1, \infty) \to \mathbb{R}$ be such that $\int_1^\infty |f(x)|\, dx < \infty$. Then $\int_1^\infty |f(x)|^2\, dx < \infty$. [1 mark].

[1] Recall that $\mathscr{C}^1(0,1)$ denotes the space of continuously differentiable functions on $(0,1)$ and $\mathscr{C}[0,1]$ denotes the space of continuous functions on $[0,1]$

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## Question 2

Suppose that $f(x) > g(x)$ for all $x \in (1, \infty)$. Prove that [2 marks].

$$\int_1^{\infty} e^{-f(x)} dx \;<\; \int_1^{\infty} e^{-g(x)} dx.$$

[We assume that both integrals in the above inequality are convergent.]

**END OF TUTORIAL QUIZ**.